PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online at
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Proposed solutions to the problems below should be submitted by May 31, 2018 via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

12013. *Proposed by David Stoner, student, Harvard University, Cambridge, MA.* Suppose that a, b, c, d, e, and f are nonnegative real numbers that satisfy $a + b + c = d + e + f$. Let t be a real number greater than 1. Prove that at least one of the inequalities

$$\begin{aligned} a^t + b^t + c^t &> d^t + e^t + f^t, \\ (ab)^t + (bc)^t + (ca)^t &> (de)^t + (ef)^t + (fd)^t, \text{ and} \\ (abc)^t &> (def)^t \end{aligned}$$

is false.

12014. *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.* Let a, b, c, and d be real numbers with $bc > 0$. Calculate

$$\lim_{n\to\infty} \begin{bmatrix} \cos(a/n) & \sin(b/n) \\ \sin(c/n) & \cos(d/n) \end{bmatrix}^n .$$

12015. *Proposed by Dao Thanh Oai, Kien Xuong, Vietnam.* Let ABC be a triangle, let G be its centroid, and let D, E, and F be the midpoints of BC, CA, and AB, respectively. For any point P in the plane of ABC, prove

$$PA + PB + PC \le 2(PD + PE + PF) + 3PG,$$

and determine when equality holds.

12016. *Proposed by Hideyuki Ohtsuka, Saitama, Japan, and Roberto Tauraso, Università di Roma "Tor Vergata," Rome, Italy.* For nonnegative integers m, n, r, and s, prove

$$\sum_{k=0}^{s} \binom{m+r}{n-k}\binom{r+k}{k}\binom{s}{k} = \sum_{k=0}^{r} \binom{m+s}{n-k}\binom{s+k}{k}\binom{r}{k}.$$

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